

String Gas Cosmology: Progress and Problems

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Abstract.

String Gas Cosmology is a model of the evolution of the very early universe based on fundamental principles and key new degrees of freedom of string theory which are different from those of point particle field theories. In String Gas Cosmology the universe starts in a quasi-static Hagedorn phase during which space is filled with a gas of highly excited string states. Thermal fluctuations of this string gas lead to an almost scale-invariant spectrum of curvature fluctuations. Thus, String Gas Cosmology is an alternative to cosmological inflation as a theory for the origin of structure in the universe. This short review focuses on the building blocks of the model, the predictions for late time cosmology, and the main problems which the model faces.

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1. Introduction

Standard Big Bang (SBB) cosmology teaches us that the further we go back in time in the history of the universe, the higher is the energy scale of the physics which is responsible for the detailed dynamical evolution of the early universe. Whereas at the present time, matter can be described by a collection of perfect fluids, already at the time of recombination atomic physics processes are crucial. Even earlier on, nuclear physics effects become important and allow the SBB to explain the origin and abundances of the light elements. Particle physics processes are crucial at even earlier times and allow us to construct models which can explain the observed asymmetry between matter and antimatter.

Neither atomic physics, nuclear physics or particle physics effects can change the conclusion that, as long as space-time is described in terms of Einstein's theory of General Relativity, the temperature and density increase to infinite values in finite time as we go back in time. This is the "Singularity Problem" of SBB cosmology. However, at sufficiently high densities it is certainly not justified to neglect quantum gravity effects. If string theory is indeed the correct theory of quantum gravity, then string theory must be essential to describe the universe at the highest densities.

The origin of structure in the universe is a mystery which cannot be explained in SBB cosmology. Given the wealth of current data on the non-random distribution of galaxies on cosmological scales and on the observed anisotropies in the temperature maps of the Cosmic Microwave Background (CMB), a crucial goal of cosmology has become to come up with a causal explanation of this data. As realized already a decade before the invention of Inflationary cosmology [1, 2], an approximately scale-invariant spectrum of primordial curvature fluctuations on scales which at early times (e.g. at the time of recombination) are super-Hubble is required to explain the observed distribution of matter on cosmological scales. These same authors realized that any model which provides such a spectrum of adiabatic fluctuations would predict angular CMB anisotropies with characteristic acoustic oscillations on angular scales smaller than about one degree.

Inflationary cosmology was proposed in 1980 [3] (see also [4] for related work) as a possible explanation for the large size, the large entropy, the approximate isotropy and the spatial flatness of the universe. It was then almost immediately realized [5] (see also [6] for related work) that inflation can provide an explanation for the origin of an approximately scale-invariant spectrum of cosmological fluctuations.

Inflationary cosmology (at least as we mostly understand it today) is based on coupling the potential energy of a slowly rolling scalar field to Einstein gravity. Thus, inflation does not eliminate the singularity problem of SBB cosmology [7]. Thus, we still need to address the question of what happened at times before the hypothetical inflationary phase. This is where string theory will play a key role.

Inflationary cosmology is faces with other conceptual problems (see e.g. [8, 9] for in depth discussions of some of these problems). For example, the wavelength of

fluctuations which we observe today is predicted to be smaller than the Planck length at the beginning of the inflationary phase in all inflation models in which the period of inflation is somewhat longer than the minimal length required for inflation to solve the problems of SBB cosmology which it is designed to solve. This is the “trans-Planckian” problem for fluctuations in inflationary cosmology [8, 10]. Furthermore, in simple models of inflation the energy scale at which inflation takes place is too close to the string scale and Planck scale to comfortably neglect quantum gravity effects. Finally, the mechanism to obtain inflationary expansion of space is subject to our ignorance on how the cosmological constant problem is solved (the driving force of inflation can be viewed as a temporary cosmological constant). In light of these problems it is useful to consider the possibility of an alternative to inflation emerging from (for example) string cosmology.

It is certainly possible that a string theoretical understanding of the very early universe will lead to a convincing realization of inflationary cosmology (see e.g. [11] for reviews of work along these lines). Most of the work on string inflation is, however, based on particle physics quantum field models motivated by string theory rather than on string theory itself, and thus may be missing crucial aspects of early universe cosmology which arise due to the specific stringy nature of matter.

String Gas Cosmology (SGC) [12] (see also [13] for related work and [14] for reviews) is a model of early universe cosmology based on making use of fundamental principles which distinguish string theory from point particle theory: the existence of new stringy degrees of freedom and the resulting new symmetries. As realized in [12], SGC provides the framework for constructing a nonsingular cosmological model. As realized more recently [15, 16], SGC leads to a mechanism for producing an approximately scale-invariant spectrum of cosmological fluctuations, and makes the key prediction [17] that the spectrum of gravitational wave should have a slight blue tilt. Thus, SGC emerges as a possible alternative to the inflationary scenario as an explanation for the observed large-scale structure of the universe.

In the following, I briefly review SGC. In the following section I present the basics of SGC. Section 3 explains how SGC leads to late time curvature fluctuations. The final section highlights challenges which SGC faces.

2. Basics of String Gas Cosmology

String Gas Cosmology is based on coupling a classical background which includes the graviton and the dilaton fields to a gas of closed strings (and possibly other basic degrees of freedom of string theory such as “branes” [18]). All dimensions of space are taken to be compact, for reasons which will become clear later. For simplicity, we take all spatial directions to be toroidal and denote the radius of the torus by R . For SGC to be effective in the way it is currently formulated, there need to be stable or at least long lived winding modes about all of the spatial dimensions which are not large today. For the SGC structure formation mechanism of [15] to work, there also need to be stable

winding modes about our three large spatial dimensions. These provide constraints on the topology of space.

Strings have three types of states: *momentum modes* which represent the center of mass motion of the string, *oscillatory modes* which represent the fluctuations of the strings, and *winding modes* counting the number of times a string wraps the torus. Since the number of string oscillatory states increases exponentially with energy, there is a limiting temperature for a gas of strings in thermal equilibrium, the *Hagedorn temperature* [19] T_H . Thus, if we take a box of strings and adiabatically decrease the box size, the temperature T will never diverge. The fact that T remains finite as R ranges from 0 to ∞ indicates that the cosmological singularity can be resolved in SGC.

The second key feature of string theory upon which SGC is based is *T-duality*. Consider the radius dependence of the energy of the basic string states: The energy of an oscillatory mode is independent of R , momentum mode energies are quantized in units of $1/R$, i.e.

$$E_n = n\mu \frac{l_s^2}{R}, \quad (1)$$

where l_s is the string length and μ is the mass per unit length of a string, but winding mode energies are quantized in units of R , i.e.

$$E_m = m\mu R, \quad (2)$$

where both n and m are integers. Thus, under the change

$$R \rightarrow 1/R \quad (3)$$

in the radius of the torus (in units of l_s) the energy spectrum of string states is invariant if winding and momentum quantum numbers are interchanged

$$(n, m) \rightarrow (m, n). \quad (4)$$

This symmetry is part of a larger group, the T-duality symmetry group. The string vertex operators are consistent with this symmetry, and thus T-duality is a symmetry of perturbative string theory. Postulating that T-duality extends to non-perturbative string theory leads [20] to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory. Specifically, T-duality will take a spectrum of stable Type IIA branes and map it into a corresponding spectrum of stable Type IIB branes with identical masses [21].

As discussed in [12], the above T-duality symmetry leads to an equivalence between small and large spaces. This reinforces the conclusion that there is no cosmological singularity in SGC. As R decreases, the physically measured size of the universe will initially decrease but then (once R has passed below the string length) increase again. The point $R = 0$ will not be reached in finite observer time.

Let us consider this evolution in a bit more detail and focus on the state of string gas matter as R changes. As R decreases from an initially very large value - maintaining thermal equilibrium - , the temperature first rises as in standard cosmology since the

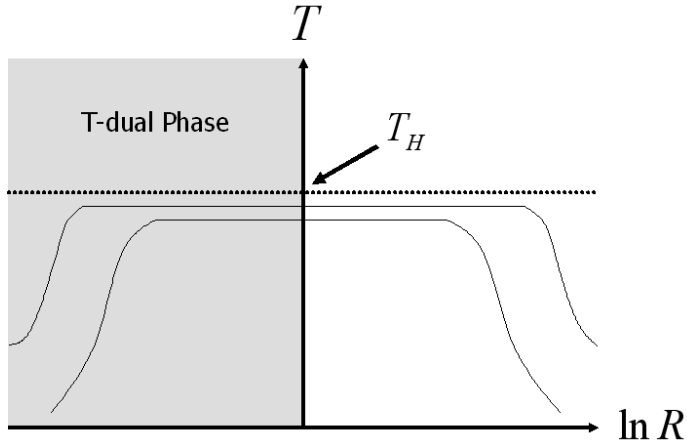


Figure 1. The temperature (vertical axis) as a function of radius (horizontal axis) of a gas of closed strings in thermal equilibrium. Note the absence of a temperature singularity. The range of values of R for which the temperature is close to the Hagedorn temperature T_H depends on the total entropy of the universe. The upper of the two curves corresponds to a universe with larger entropy.

string states which are occupied (the momentum modes) get heavier. However, as the temperature approaches the Hagedorn temperature, the energy begins to flow into the oscillatory modes and the increase in temperature levels off. As the radius R decreases below the string scale, the temperature begins to decrease as the energy begins to flow into the winding modes whose energy decreases as R decreases (see Figure 1).

The equations that govern the background of string gas cosmology are not known. The Einstein equations are not the correct equations since they do not obey the T-duality symmetry of string theory. Many early studies of string gas cosmology were based on using the dilaton gravity equations [22, 23, 24], However, these equations are not satisfactory, either, as will be discussed in Section 4.

Some conclusions about the time-temperature relation in string gas cosmology can be derived based on thermodynamical considerations alone. One possibility is that R starts out much smaller than the self-dual value and increases monotonically. From Figure 1 it then follows that the time-temperature curve will correspond to that of a bouncing cosmology. Alternatively, it is possible that the universe starts out in a metastable state near the Hagedorn temperature, the *Hagedorn phase*, and then smoothly evolves into an expanding phase dominated by radiation like in standard cosmology (Figure 2). SGC is based on this assumption. Note that we are assuming that not only the scale factor but also the dilaton is constant in time.

The transition between the quasi-static Hagedorn phase and the radiation phase at the time t_R is a consequence of the annihilation of string winding modes into string loops. Since this process corresponds to the production of radiation, we denote this time by the same symbol t_R as the time of reheating in inflationary cosmology. As pointed out in [12], this annihilation process only is possible in at most three large spatial dimensions. This is a simple dimension counting argument: string world sheets have measure zero

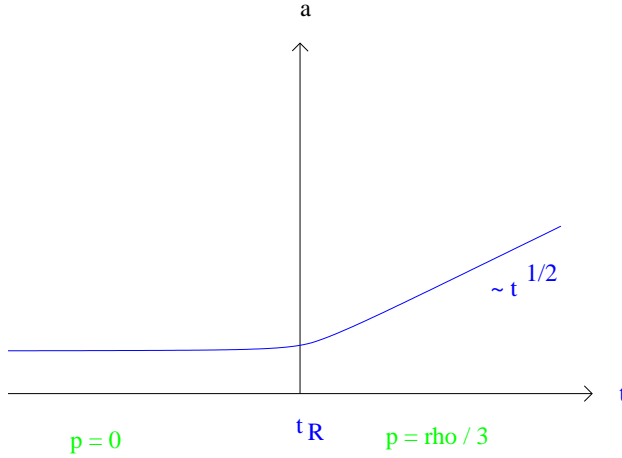


Figure 2. The dynamics of string gas cosmology. The vertical axis represents the scale factor of the universe, the horizontal axis is time. Along the horizontal axis, the approximate equation of state is also indicated. During the Hagedorn phase the pressure is negligible due to the cancellation between the positive pressure of the momentum modes and the negative pressure of the winding modes, after time t_R the equation of state is that of a radiation-dominated universe.

intersection probability in more than four large space-time dimensions. Hence, string gas cosmology may provide a natural mechanism for explaining why there are exactly three large spatial dimensions.

One of the important features of SGC is that there is a natural mechanism to stabilize the size and shape moduli of the extra spatial dimensions, at least in heterotic string theory. The early Hagedorn phase of SGC is characterized by a gas in which all string states are excited. This includes modes with both winding and momentum numbers about the extra spatial dimensions. In heterotic string theory the lowest energy state are “enhanced symmetry states” which have zero mass when the radius of the extra spatial section equals the string scale, but non-vanishing winding and momenta. Following initial work of [25] and [26] (see also [27]), size moduli stabilization was studied in detail in [28, 29], and shape moduli stabilization in [30]. The only modulus which is not stabilized using the basic ingredients of SGC is the axion-dilaton modulus. As studied in [31], this modulus can be stabilized by making use of gaugino condensation, which in turn also leads to supersymmetry breaking [32] which is consistent with the current cosmological constraints. The gaugino condensation mechanism of dilaton stabilization does not disrupt the natural stabilization of the size and shape moduli via string gas effects [31].

The size modulus stabilization mechanism [25, 26, 28] is very geometrical: states with non-vanishing momenta and windings about the extra dimensions lead to a force resisting both contraction (because then the momenta lead to a large mass) and expansion (because then the winding states get very heavy), thus yielding a preferential value of the radius which is given by the string scale (larger if there is a chemical potential for winding number). In the case of heterotic string theory, the lightest states

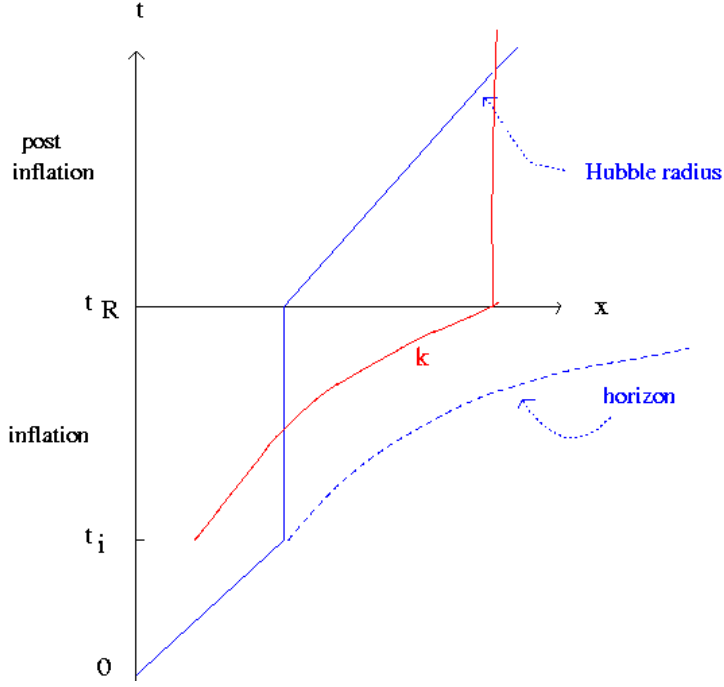


Figure 3. Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis corresponds to physical distance. The solid line labelled k is the physical length of a fixed comoving fluctuation scale. The role of the Hubble radius and the horizon are discussed in the text.

for a value of the size modulus close to the string scale are the enhanced symmetry states which contain both windings and momenta and whose energy at a value of the radion equal to the string length is given by the state's kinetic energy in the large dimensions. As studied carefully in [29], these states fix the radion in a way which is consistent with cosmological constraints. The extra states act as radiation from our four-dimensional space-time point of view. With very little tuning of the initial density of the enhanced symmetry states, the resulting contribution of the states to the current radiation density is below the nucleosynthesis constraints, and at the same time the mass of the radion fluctuations about the ground state is above the current lower bound.

3. Cosmological Fluctuations from String Gas Cosmology

In this section we show how thermal fluctuations in the Hagedorn phase of SGC lead to an almost scale-invariant spectrum of cosmological perturbations. It is useful to first remind the reader of the mechanism by which inflationary cosmology leads to a causal generation mechanism for cosmological fluctuations yielding an almost scale-invariant spectrum of perturbations. The space-time diagram of inflationary cosmology is sketched in Figure 3.

During the period of inflation, the Hubble radius $l_H(t) \equiv \frac{a}{\dot{a}}$ is approximately constant. In contrast, the physical length of a fixed co-moving scale (labelled by k in the figure) is expanding exponentially. In this way, in inflationary cosmology scales which

have microscopic sub-Hubble wavelengths at the beginning of inflation are red-shifted to become super-Hubble-scale fluctuations at the end of the period of inflation. In the post-inflationary phase of Standard cosmology the Hubble radius increases linearly in time, i.e. faster than the physical wavelength corresponding to a fixed co-moving scale. Thus, scales re-enter the Hubble radius at late times.

Since inflation red-shifts any classical fluctuations which might have been present at the beginning of the inflationary phase, fluctuations in inflationary cosmology are taken to be generated by quantum vacuum perturbations [5]. The fluctuations begin in their quantum vacuum state at the onset of inflation. Once the wavelength exceeds the Hubble radius, squeezing of the wave-function of the fluctuations sets in (see [33, 34] for reviews of the theory of cosmological perturbations). This squeezing plus the decoherence of the fluctuations due to the interaction between short and long wavelength modes generated by the intrinsic non-linearities in both the gravitational and matter sectors of the theory (see [35, 36] for recent discussions of this aspect and references to previous work) lead to the classicalization of the fluctuations on super-Hubble scales.

The process of generation and evolution of cosmological fluctuations in SGC is very different. Recall that the evolution of the scale factor in SGC is as represented in Figure 2 and leads to a space-time diagram as is sketched in Figure 4. As in Figure 3, the vertical axis is time and the horizontal axis denotes the physical distance. For times $t < t_R$, we are in the static Hagedorn phase and the Hubble radius is infinite. For $t > t_R$, the Einstein frame Hubble radius is expanding as in standard cosmology. The time t_R is when the string winding modes begin to decay into string loops, and the scale factor starts to increase, leading to the transition to the radiation phase of standard cosmology.

First of all, we see that in SGC all fluctuations emerge at early times from sub-Hubble scales, as in inflationary cosmology. This is the first key requirement for a causal generation mechanism to be possible. Scales are existing the Hubble radius not because their wavelength is increasing (as in the case of inflation), but rather because the Hubble radius is decreasing during the transition between the Hagedorn phase and the expanding radiation phase. Since the physical wavelength of fluctuations is constant in the Hagedorn phase, the Trans-Planckian problem for fluctuations does not arise: if we follow scale which corresponds to the current Hubble radius back to the time t_R , then - taking the Hagedorn temperature to be of the order 10^{16} GeV - we obtain a length of about 1 mm. Compared to the string scale and the Planck scale, this is in the far infrared.

The fact that the Hagedorn phase is static and dominated by a gas of strings lead to a generation mechanism for fluctuations which is very different from that in inflationary cosmology. Instead of quantum vacuum perturbations we have thermal fluctuations, and thermal fluctuations not of a particle gas, but of a gas of strings. It was realized in [15] that this mechanism yields approximately scale-invariant curvature perturbations at late times.

The main steps in the computation of cosmological fluctuations in SGC is as

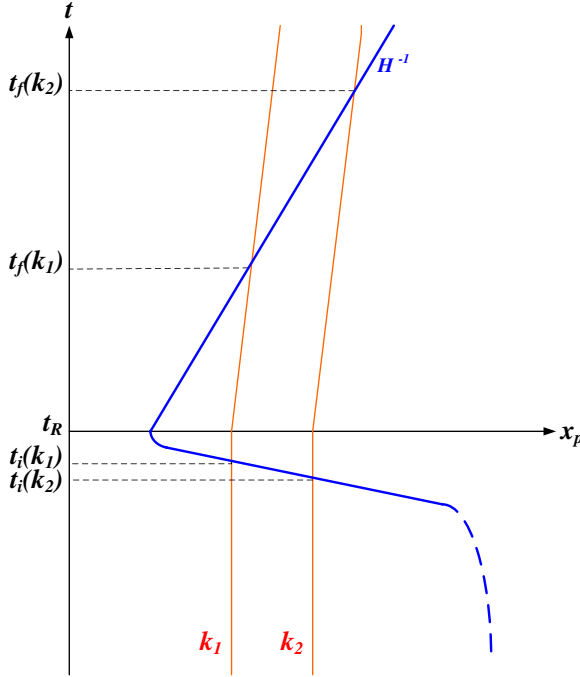


Figure 4. Space-time diagram (sketch) showing the evolution of fixed co-moving scales in string gas cosmology. The vertical axis is time, the horizontal axis is physical distance. The solid curve represents the Einstein frame Hubble radius H^{-1} which shrinks abruptly to a micro-physical scale at t_R and then increases linearly in time for $t > t_R$. Fixed co-moving scales (the dotted lines labeled by k_1 and k_2) which are currently probed in cosmological observations have wavelengths which are smaller than the Hubble radius before t_R . They exit the Hubble radius at times $t_i(k)$ just prior to t_R , and propagate with a wavelength larger than the Hubble radius until they reenter the Hubble radius at times $t_f(k)$.

follows [15] (see [16] for a more detailed description). For a fixed co-moving scale with wavenumber k we compute the matter fluctuations while the scale is sub-Hubble (and therefore gravitational effects are sub-dominant). When the scale exits the Hubble radius at time $t_i(k)$ we use the gravitational constraint equations to determine the induced metric fluctuations, which are then propagated to late times using the usual equations of gravitational perturbation theory. Since the scales we are interested in are in the far infrared, we use the Einstein constraint equations for fluctuations.

We write the metric including cosmological perturbations (scalar metric fluctuations) and gravitational waves in the following form (using conformal time η which is related to the physical time t via $dt = a(t)d\eta$)

$$ds^2 = a^2(\eta) \left\{ (1 + 2\Phi)d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j \right\}. \quad (5)$$

Here, we have adopted the longitudinal gauge and we have taken matter to be free of anisotropic stress. The spatial tensor $h_{ij}(\mathbf{x}, t)$ is transverse and traceless and represents gravitational waves.

Note that in contrast to the case of slow-roll inflation, scalar metric fluctuations and gravitational waves are generated by matter at the same order in cosmological perturbation theory. This could lead to the expectation that the amplitude of gravitational waves in string gas cosmology should be generically larger than in inflationary cosmology. This expectation, however, is not realized [17] since there is a different mechanism which suppresses the gravitational waves relative to the density perturbations (namely the fact that the gravitational wave amplitude is set by the amplitude of the pressure, and the pressure is suppressed relative to the energy density in the Hagedorn phase).

Assuming that the fluctuations are described by the perturbed Einstein equations (they are *not* if the dilaton is not fixed [37, 38]), then the spectra of cosmological perturbations Φ and gravitational waves h are given by the energy-momentum fluctuations in the following way [16]

$$\langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle, \quad (6)$$

where the pointed brackets indicate expectation values, and

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle, \quad (7)$$

where the right hand side of (7) is the average over the correlation functions with $i \neq j$, and h is the amplitude of the gravitational waves ‡.

To determine the spectrum of scalar metric fluctuations, we first calculate the root mean square energy density fluctuations in a sphere of radius $R = k^{-1}$. For a system in thermal equilibrium they are given by the specific heat capacity C_V via

$$\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V. \quad (8)$$

The specific heat of a gas of closed strings on a torus of radius R can be derived from the partition function of a gas of closed strings. This computation was carried out in [39] and yields

$$C_V \approx 2 \frac{R^2 / \ell^3}{T(1 - T/T_H)}. \quad (9)$$

The specific heat capacity scales holographically with the size of the box. This result follows rigorously from evaluating the string partition function in the Hagedorn phase. The result, however, can also be understood heuristically: in the Hagedorn phase the string winding modes are crucial. These modes look like point particles in one less spatial dimension. Hence, we expect the specific heat capacity to scale like in the case of point particles in one less dimension of space §.

‡ The gravitational wave tensor h_{ij} can be written as the amplitude h multiplied by a constant polarization tensor.

§ We emphasize that it was important for us to have compact spatial dimensions in order to obtain the winding modes which are crucial to get the holographic scaling of the thermodynamic quantities.

With these results, the power spectrum $P(k)$ of scalar metric fluctuations can be evaluated as follows

$$\begin{aligned}
P_\Phi(k) &\equiv \frac{1}{2\pi^2} k^3 |\Phi(k)|^2 \\
&= 8G^2 k^{-1} \langle |\delta\rho(k)|^2 \rangle \\
&= 8G^2 k^2 \langle (\delta M)^2 \rangle_R \\
&= 8G^2 k^{-4} \langle (\delta\rho)^2 \rangle_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H},
\end{aligned} \tag{10}$$

where in the first step we have used (6) to replace the expectation value of $|\Phi(k)|^2$ in terms of the correlation function of the energy density, and in the second step we have made the transition to position space

The first conclusion from the result (10) is that the spectrum is approximately scale-invariant ($P(k)$ is independent of k). It is the ‘holographic’ scaling $C_V(R) \sim R^2$ which is responsible for the overall scale-invariance of the spectrum of cosmological perturbations. However, there is a small k dependence which comes from the fact that in the above equation for a scale k the temperature T is to be evaluated at the time $t_i(k)$. Thus, the factor $(1 - T/T_H)$ in the denominator is responsible for giving the spectrum a slight dependence on k . Since the temperature slightly decreases as time increases at the end of the Hagedorn phase, shorter wavelengths for which $t_i(k)$ occurs later obtain a smaller amplitude. Thus, the spectrum has a slight red tilt.

Now let us turn to the key prediction with which SGC can be differentiated observationally from inflation. It concern the tilt in the spectrum of gravitational waves. As discovered in [17], the spectrum of gravitational waves is also nearly scale invariant. However, in the expression for the spectrum of gravitational waves the factor $(1 - T/T_H)$ appears in the numerator, thus leading to a slight blue tilt in the spectrum. This contrasts with the predictions of inflationary models, where quite generically a slight red tilt for gravitational waves results. The physical reason for the blue tilt of the spectrum of gravitational waves in SGC is that large scales exit the Hubble radius earlier when the pressure and hence also the off-diagonal spatial components of $T_{\mu\nu}$ are closer to zero.

The method for calculating the spectrum of gravitational waves follows what was presented above for the scalar metric fluctuations. For a mode with fixed co-moving wavenumber k , we compute the correlation function of the off-diagonal spatial elements of the string gas energy-momentum tensor at the time $t_i(k)$ when the mode exits the Hubble radius and use (7) to infer the amplitude of the power spectrum of gravitational waves at that time. We then follow the evolution of the gravitational wave power spectrum on super-Hubble scales for $t > t_i(k)$ using the equations of general relativistic perturbation theory.

The power spectrum of the tensor modes is given by (7):

$$P_h(k) = 16\pi^2 G^2 k^{-4} k^3 \langle \delta T^i_j(k) \delta T^i_j(k) \rangle \tag{11}$$

for $i \neq j$. Note that the k^3 factor multiplying the momentum space correlation function of T_j^i gives the position space correlation function $C_{j\ j}^i{}^i(R)$, namely the root mean square fluctuation of T_j^i in a region of radius $R = k^{-1}$ (the reader who is skeptical about this point is invited to check that the dimensions work out properly). Thus,

$$P_h(k) = 16\pi^2 G^2 k^{-4} C_{j\ j}^i{}^i(R). \quad (12)$$

The correlation function $C_{j\ j}^i{}^i$ on the right hand side of the above equation follows from the thermal closed string partition function and was computed explicitly in [40] (see also [16]). We obtain

$$P_h(k) \sim 16\pi^2 G^2 \frac{T}{l_s^3} (1 - T/T_H) \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - T/T_H) \right], \quad (13)$$

which, for temperatures close to the Hagedorn value reduces to

$$P_h(k) \sim \left(\frac{l_{Pl}}{l_s} \right)^4 (1 - T/T_H) \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - T/T_H) \right]. \quad (14)$$

This shows that the spectrum of tensor modes is - to a first approximation, namely neglecting the logarithmic factor and neglecting the k -dependence of $T(t_i(k))$ - scale-invariant.

On super-Hubble scales, the amplitude h of the gravitational waves is constant. The wave oscillations freeze out when the wavelength of the mode crosses the Hubble radius. As in the case of scalar metric fluctuations, the waves are squeezed. Whereas the wave amplitude remains constant, its time derivative decreases. Another way to see this squeezing is to change variables to

$$\psi(\eta, \mathbf{x}) = a(\eta) h(\eta, \mathbf{x}) \quad (15)$$

in terms of which the action has canonical kinetic term. The action in terms of ψ becomes

$$S = \frac{1}{2} \int d^4x \left(\psi'^2 - \psi_{,i} \psi_{,i} + \frac{a''}{a} \psi^2 \right) \quad (16)$$

from which it immediately follows that on super-Hubble scales $\psi \sim a$. This is the squeezing of gravitational waves [41]. Since there is no k -dependence in the squeezing factor, the scale-invariance of the spectrum at the end of the Hagedorn phase will lead to a scale-invariance of the spectrum at late times.

Note that in the case of string gas cosmology, the squeezing factor $z(\eta)$ does not differ substantially from the squeezing factor $a(\eta)$ for gravitational waves. In the case of inflationary cosmology, $z(\eta)$ and $a(\eta)$ differ greatly during reheating, leading to a much larger squeezing for scalar metric fluctuations, and hence to a suppressed tensor to scalar ratio of fluctuations. In the case of string gas cosmology, there is no difference in squeezing between the scalar and the tensor modes.

Let us return to the discussion of the spectrum of gravitational waves. The result for the power spectrum is given in (14), and we mentioned that to a first approximation this corresponds to a scale-invariant spectrum. As realized in [17], the logarithmic term

and the k -dependence of $T(t_i(k))$ both lead to a small blue-tilt of the spectrum. This feature is characteristic of our scenario and cannot be reproduced in inflationary models. In inflationary models, the amplitude of the gravitational waves is set by the Hubble constant H . The Hubble constant cannot increase during inflation, and hence no blue tilt of the gravitational wave spectrum is possible.

A heuristic way of understanding the origin of the slight blue tilt in the spectrum of tensor modes is as follows. The closer we get to the Hagedorn temperature, the more the thermal bath is dominated by long string states, and thus the smaller the pressure will be compared to the pressure of a pure radiation bath. Since the pressure terms (strictly speaking the anisotropic pressure terms) in the energy-momentum tensor are responsible for the tensor modes, we conclude that the smaller the value of the wavenumber k (and thus the higher the temperature $T(t_i(k))$ when the mode exits the Hubble radius, the lower the amplitude of the tensor modes. In contrast, the scalar modes are determined by the energy density, which increases at $T(t_i(k))$ as k decreases, leading to a slight red tilt.

The reader may ask about the predictions of string gas cosmology for non-Gaussianities. The answer is [42] that the non-Gaussianities from the thermal string gas perturbations are Poisson-suppressed on scales larger than the thermal wavelength in the Hagedorn phase. However, if the spatial sections are initially large, then it is possible that a network of cosmic superstrings [43] will be left behind. These strings - if stable - would achieve a scaling solution (constant number of strings crossing each Hubble volume at each time). Such strings give rise to linear discontinuities in the CMB temperature maps [44], lines which can be searched for using edge detection algorithms such as the Canny algorithm (see [45] for recent feasibility studies).

4. Challenges for String Gas Cosmology

As has been argued in the above sections, SGC is an interesting model of early universe cosmology making use of fundamental principles of string theory which are not used in the standard string-motivated field theory approaches to string cosmology (most of string inflation model building falls into that category). SGC realized the hope that string theory will lead to a non-singular cosmology since there is a duality between large and small spatial sections [12]. SGC provides a natural solution to the stabilization of size and shape moduli [29]: these moduli are stabilized without the need to add any extra ingredients to the model. One extra ingredient is required in order to stabilize the dilaton - gaugino condensation is one possibility [31].

Thermal fluctuations of the string gas during an early quasi-static Hagedorn phase lead to an almost scale-invariant power spectrum of cosmological perturbations. Thus, SGC is an alternative to cosmological inflation for explaining the origin of structure in the universe. The key prediction of SGC is a slight blue tilt in the spectrum of gravitational waves, whereas inflation generically produces a slight red tilt.

The main problem of the current implementation of SGC is that it does not provide

a quantitative model for the Hagedorn phase. This phase must be quasi-static (including a stable dilaton) and last for a sufficient length of time to be able to set up thermal equilibrium on scales which today become comparable to the current Hubble radius (if the string scale is close to the scale of Grand Unification), this length is of the order of 1mm). The criticisms of string gas cosmology raised in [38, 46] are based on assuming that the background is described by dilaton gravity.

The problem is that we currently have no framework for describing the Hagedorn phase mathematically. The Einstein action is obviously inapplicable since it conflicts with the T-duality symmetry which is crucial to string theory. Neither can dilaton gravity provide suitable background equations. Firstly, as the dilaton changes from weak to strong coupling, the nature of the light states in the string spectrum changes and it becomes inconsistent to model matter as a gas of strings. More importantly, at high densities such as the Hagedorn density the Einstein term in the gravitational action of both Einstein and dilaton gravity will no longer be the dominant terms in a derivative expansion of the action. This is known from all approaches to quantum gravity. Thus, we cannot expect that a simple effective action such as that of dilaton gravity will apply. Nevertheless, to make the string gas cosmology scenario into a real theory, it is crucial to obtain a good understanding of the background dynamics. For some initial steps in this direction see [47]. Another study of this problem was given in [48].

A second problem of string gas cosmology is the size problem (and the related entropy problem). If the string scale is about 10^{17}GeV as is preferred in early heterotic superstring models, then the radius of the universe during the Hagedorn phase must be many orders of magnitude larger than the string scale. Without embedding string gas cosmology into a bouncing cosmology, it seems unnatural to demand such a large initial size. This problem disappears if the Hagedorn phase is preceded by a phase of contraction, as in the model of [49]. In this case, however, it is non-trivial to arrange that the Hagedorn phase lasts sufficiently long to maintain thermal equilibrium over the required range of scales.

It should be noted, however, that some of the conceptual problems of inflationary cosmology such as the trans-Planckian problem for fluctuations, do not arise in string gas cosmology. As in the case of the matter bounce scenario, the basic mechanism of the scenario is insensitive to what sets the cosmological constant to its observed very small value.

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